I. INTRODUCTION

Scattering of an atomic beam by a standing laser wave has been a subject of interest for a long time [1,2]. In analogy to the diffraction of electrons by a crystal lattice, the optical standing wave acts as a thick grating, which is capable of splitting the incident beam of atoms. The optical lattice, however, offers additional possibilities and allows the study of effects never observed in experiments with solid-state systems. In the present paper we study the scattering of atoms from a "dynamical light lattice," which can be realized by modulating the laser intensity. We shall show that this case involves a specific scattering regime, which is explained in terms of quantum chaos [3–5].

The structure of the paper is the following. In Sec. II we recall some results on atomic diffraction in the case of no modulation. We define the model and briefly discuss the different regimes of scattering depending on the system parameters. We also specify the region of the parameters for which the analysis of the system dynamics in the case of a modulated standing wave will be carried out. This region corresponds to a large intensity of the laser field, far beyond the perturbation limit. As shown in Sec. II A, this case meets the semiclassical condition. Thus the results of the classical analysis can be applied to analyze and interpret the quantum dynamics. We shall make use of classical mechanics throughout the paper.

The main results are reported in Sec. III. We shall consider the case of a sinusoidal modulation of the field amplitude. Then, from the viewpoint of modern nonlinear mechanics, our system of interest is the system of three nonlinear interacting resonances. Depending on the modulation frequency, it shows either quasiregular or chaotic dynamics. It should be pointed out here that the system of interacting resonances is one of the basic models for classical Hamiltonian chaos [5–7]. It is no wonder that it has attracted much attention in connection with the problem of quantum chaos. In particular, the dynamics of the system of two and three interacting resonances was studied in Refs. [8–10]. In the present paper we extend these studies to the problem of atomic scattering by a laser wave. We show that in the case of sinusoidal modulation of the laser field the atomic Bragg scattering offers an interesting realization of the system of three interacting quantum resonances, enabling a laboratory study [11]. In contrast, the realization of the regime of interacting resonances suggests specific regimes of the Bragg scattering that were not observed before.

II. SCATTERING BY A LIGHT LATTICE

We consider the scattering of an atomic beam slowly passing through a laser beam. In this section the intensity of laser radiation is assumed to be constant. We shall refer to this case as scattering by a "rigid" light lattice.

A. The model and parameters

We introduce the model most frequently used in the context of atomic scattering: The atom is approximated by a single parameter. To show this, we introduce the frequency 

\[ \Omega_{\text{rec}} = \frac{\hbar k^2}{2m}, \]

In Eq. (1) the profile function \( \epsilon(t) \) models the finite time of the interaction between the atom and the field, where the characteristic "switching" time \( \tau \) is the width of the laser beam divided by the longitudinal component of atom velocity. To be specific, we choose a Gaussian profile \( \epsilon(t) = \exp[-(t/\tau)^2] \).

The classical dynamics of the system (1) depends only on a single parameter. To show this, we introduce the frequency

\[ \Omega_{\text{ph}} = (\omega_{\text{rec}} \Omega_{\text{eff}})^{1/2}, \quad \omega_{\text{rec}} = \frac{\hbar k^2}{2m}, \]

Thus the results of the classical analysis can be applied to analyze and interpret the quantum dynamics. We shall make use of classical mechanics throughout the paper.
which is the frequency of small oscillations near the bottom of the light potential in the center of the laser beam, and rescale the variables on the basis of this frequency and the field wave number k: \( x' = 2kx, t' = \Omega_{ph}t, \quad \omega' = \Omega_{ph}\tau, \quad p' = p(2k/\Omega_{ph}m), \) and \( H' = 4H/\Omega_{eff} \). Then, up to an additive term, the Hamiltonian (1) takes the form

\[
H' = p'^2/2 - \exp[-(t'/\tau')^2] \cos x'.
\]

The Hamiltonian (3) contains only one parameter: the passage time \( \tau' \). In what follows we restrict ourselves to the adiabatic case \( \tau' \gg 2\pi \) or \( \tau' \gg 2\pi/\Omega_{ph} \) (a rigorous definition of the adiabatic regime is given below).

In the quantum case, Planck’s constant is also affected by the scaling and appears in a dimensionless scaled form

\[
\hbar' = 4\Omega_{ph}/\Omega_{eff} = 4 \left( \frac{2\omega_{rec}}{\Omega_{eff}} \right)^{1/2}.
\]

The value of this effective \( \hbar' \) defines both the strength of the interaction of the atom with the light beam and the degree of “classicality.” The quantum regimes \( \hbar' > 1 \) and \( \hbar' \sim 1 \) correspond to weak and moderate interactions. These regimes are analyzed, for example, in [13]. (We note that the adiabatic condition \( \tau' \gg 2\pi/\Omega_{ph} \) reads \( \tau' \gg \omega_{rec}^{-1} \) in this case.) The semiclassical regime \( \hbar' < 1 \) has been analyzed in a previous paper [14], the main results of which are briefly summarized in the following subsection.

B. Adiabatic scattering

First we discuss the scattering of a classical atom. Rigorously, the adiabatic regime corresponds to conservation of the action integral

\[
I = \frac{1}{2\pi} \int p'dx' = \frac{1}{\pi} \int_{x_{min}}^{x_{max}} p'dx'.
\]

The integral (5) is conserved if the rate of change of the potential (defined by the parameter \( \tau \)) is small relative to the characteristic frequency of the system dynamics (defined by the frequency \( \Omega_{ph} \)), i.e., \( \tau \Omega_{ph} \gg 2\pi \) [15]. Since \( I(t' = \pm \infty) = 2p'(t' = \pm \infty), \) it follows from conservation of the action integral that the absolute values of the initial \( p'_{in} \) and final \( p' \) momentum of the classical atom are equal. The sign, however, can change if \( |p'_{in}| < p'_{max} \), with

\[
p'_{max} = \frac{1}{2\pi} \int_{-\pi}^{\pi} (2 - 2\cos x')^{1/2} dx' = \frac{4}{\pi}
\]

(\( 2p'_{max} \) corresponds to the action for the separatrix trajectory of a particle in the potential cosine). This phenomenon can be understood by noting that, provided \( |p'_{in}| < p'_{max} \), the atom enters the vibrational regime of motion for \( t' \) close to zero. Thus the beam of classical atoms is split into two beams with \( p' = \pm p'_{in} \) for \( |p'_{in}| < p'_{max} \), whereas it is not scattered in the opposite case [14].

The quantum dynamics of the system (3) for \( \hbar' < 1 \) resembles the classical dynamics in the sense that an atom, initially in the state \( p'_{in} \), is scattered into the state \( p' = -p'_{in} \) if \( |p'_{in}| < p'_{max} \) and is practically not scattered if \( |p'_{in}| > p'_{max} \) (see Fig. 5 in Ref. [14]). A deviation from the classical result is a consequence of the discrete nature of the transitions between quantum states: The atom can change its momentum only by an even multiple of the photon momentum. The discreteness of the transitions manifests in two effects: First, scattering is suppressed for \( p'_{in} \neq \hbar'/2 \) and \( l \) is an integer and, second, the probability of scattering shows an oscillation (the so-called Pendëllosung oscillation [13]) when a parameter of the system is varied.

The formal analysis of the problem uses the instantaneous eigenfunctions of the Hamiltonian (1), which are the Mathieu functions. (An important characteristic of the adiabatic regime is that only two instantaneous eigenfunctions are involved in the analysis.) In this way, an estimate for the width of the diffraction peaks can be obtained:

\[
\Delta p'_{f} \sim \hbar'/\tau' p'_{f}, \quad \Delta p'_{i} = \hbar'/2.
\]

It is useful to note that the estimate (7) can also be found in a qualitative manner. In fact, for large values of \( \tau \), the conservation of the energy holds with accuracy \( E'_{in} - E'_{f} \sim \hbar'/\tau', E'_{f} = p'^2/2. \) Bearing in mind the conservation of the momentum \( p'_{f} - p'_{in} = \hbar'/l \), we immediately obtain Eq. (7).

III. SCATTERING BY A “DYNAMICAL” LIGHT LATTICE

We proceed with the case of a laser amplitude that is periodically modulated in time. To be specific, we shall consider a sinusoidal modulation. Then the Hamiltonian (3) takes the form

\[
H' = p'^2/2 - \varepsilon(t')\cos^2(\omega t') \cos x',
\]

\[
\varepsilon(t') = \exp[-(t'/\tau')^2],
\]

i.e., the laser intensity changes proportionally to \( \cos^2(\omega t) \). In what follows we omit the primes for the scaled variables.

A. Interaction of nonlinear resonances

First we discuss the classical dynamics of the system (8) for fixed \( \varepsilon \). It is convenient to rewrite the Hamiltonian (8) in the form

\[
H = p'^2/2 - (\varepsilon/2)\cos x - (\varepsilon/4)\cos(x + 2\omega t) - (\varepsilon/4)\cos(x - 2\omega t).
\]

The last three terms on the right-hand side of Eq. (9) cause three nonlinear resonances located at \( p = 0 \) and \( p = \pm 2\omega \) [see Fig. 1(a)]. In the central resonance at \( p = 0 \) the atoms are fixed, whereas in the upper \( p = 2\omega \) or lower \( p = -2\omega \) resonance they move in the positive or negative direction of the laser beam, i.e., to the right or to the left. The width of the resonances can be characterized by their separatrix momentum \( p_{max} = (4/\pi)^{1/2} \varepsilon^{1/2} \) for the central resonance and \( p_{max} = (4/\pi)^{1/2} \varepsilon^{1/4} \) for the side resonances. When the widths of the resonances are small compared to their distance, i.e., for \( \varepsilon \)
one can neglect their mutual influence and the dynamics of the system in the vicinity of any of the three nonlinear resonances is governed by the corresponding term in Eq. (9). For example, in the vicinity of the right resonance, the effective Hamiltonian is \( H_{\text{eff}} = p^2/2 - (\epsilon/4)\cos(x - 2\omega t) \). After a canonical substitution \( p = p - 2\omega \), \( x = x - 2\omega t \) it reduces to the form

\[
\tilde{H}_{\text{eff}} = \tilde{p}^2/2 - (\epsilon/4)\cos\tilde{x},
\]

which coincides with the one considered in Sec. II.

By decreasing of the modulation frequency \( \omega \), the mutual influence of the resonances becomes stronger. This leads to the appearance of secondary nonlinear resonances and chaotic layers substituting the separatrix of the resonances and eventually to the development of global chaos for \( K > 1 \) [see Fig. 1(b)]. From the definition of \( K \) in Eq. (10), the transition to chaos can also be considered to be caused by an increase of \( \epsilon \) for a fixed value of the driving frequency. To distinguish these two cases we shall refer to the parameter (10) as either \( K = K(\omega) \) or \( K = K(\epsilon) \).

We would like to emphasize that the system (8) has no other global integral of motion except the quasienergy

\[
E = \omega J + p^2/2 - \epsilon\cos^2\phi \cos x
\]

(\( \phi = \omega t \) is the phase of the field and \( J \) is a variable conjugate to \( \phi \)). However, for \( K < 1 \) the system possesses local integrals of motion, which are defined only in a part of the phase space. For example, for the right primary resonance in Fig. 2(a), the local integrals are

\[
G = J + 2p, \quad I = \frac{1}{2\pi} \int p \, d\tilde{x}.
\]

Because of the local character of the integrals, the system is called almost integrable or quasiregular.

---

**FIG. 1.** Phase portrait of the system (8) for \( \epsilon = 1 \) and (a) \( \omega = 2.9 \) and (b) \( \omega = 0.75 \).

**FIG. 2.** Distribution of (transversal) momenta of the outgoing atomic beam. [The distribution is plotted against the scaled momentum; in the unscaled variables the distance between neighboring peaks in (a) and (c) equals \( \hbar k \).] The parameters are \( \omega = 2.9 \), \( \tau = 16\pi \), and \( \hbar = 0.1 \). The incoming beam was chosen to have a Gaussian distribution with mean momentum (a) \( \langle p_{\text{in}} \rangle = 1 \), (b) \( \langle p_{\text{in}} \rangle = 3 \), and (c) \( \langle p_{\text{in}} \rangle = 5 \). The vertical lines show the regions of influence of the primary (classical) resonances.

We proceed with the case \( \epsilon(t) = \exp[-(t/\tau)^2] \). Two different regimes should be distinguished depending on the driving frequency \( \omega \): case (i), where the dynamics of the system is quasiregular for any \( \epsilon \) in the interval \([0,1]\), and case (ii), where the system enters a chaotic regime for \( \epsilon \) larger than some critical \( \epsilon_{\text{cr}} \), i.e., \( K(\epsilon > \epsilon_{\text{cr}}) > 1 \). We discuss the dynamics in these regions from the viewpoint of quantum mechanics in the following sections.

**B. Scattering by nonlinear resonances**

Figure 2 shows the results of a numerical simulation of the scattering of an atomic beam by an amplitude-modulated laser beam for \( \omega = 2.9 \), \( \hbar = 0.1 \), and \( \tau = 16\pi \). The momentum distribution of the incoming beam was chosen to be Gaussian with nonzero mean momentum. (In a laboratory experiment, the mean momentum of the incoming beam is usually varied by changing the crossing angle between the atomic and the laser beam.) Three values of the mean momentum are considered: \( \langle p_{\text{in}} \rangle = 1 \) [Fig. 2(a)], \( \langle p_{\text{in}} \rangle = 3 \) [Fig. 2(b)], and \( \langle p_{\text{in}} \rangle = 5 \).
= 5 [Fig. 2(c)]. These three cases illustrate the scattering by the central primary nonlinear resonance, by the secondary resonance, and by the right primary resonance [see Fig. 1(a)]. The vertical lines in the figures indicate the regions of influence of the primary resonances, given by \( p_{\text{max}} \) [17].

As is expected from Eqs. (11) and (13), scattering from the primary nonlinear resonances resembles that for the case of a rigid light lattice. The main difference is that the positions of the diffraction peaks in Fig. 2 depend on the value of the driving frequency. (If we slightly change the frequency \( \omega \), the distribution moves as a whole without changing its shape.) This can be easily shown by a qualitative estimate based on approximate conservation of the quasienergy (12):

\[
E_f - E_m = \omega J + \frac{p_f^2}{2} - \omega J_m - \frac{p_m^2}{2} \sim \frac{\hbar}{\tau}.
\]

In fact, in the vicinity of the right resonance, the system has the local integral \( G = J + 2p \). Then Eq. (14) takes the form \((p_f - 2\omega)^2/2 - (p_m - 2\omega)^2/2 \sim \hbar/\tau\). Using the conservation of the momentum \( p_f - p_m = \hbar l \), we see that scattering is efficient only for

\[
p_f = 2\omega + \hbar l/2, \quad \Delta p_l \sim 1/\hbar l.
\]

It should be noted that \( \omega = 2.9 \) corresponds to a rather weak interaction of the primary resonances. Because of this, the shape of the curves shown in Figs. 2(a) and 2(c) remains practically the same for larger values of the driving frequency. This not the case, however, for a secondary nonlinear resonance [Fig. 2(b)]. The largest secondary nonlinear resonance is situated in the middle between the primary nonlinear resonances and the system dynamics in its vicinity is governed by the effective Hamiltonian

\[
\tilde{H}_{\text{eff}} = p^2/2 - c\cos(2\tilde{x}).
\]

Unlike in the case of a primary resonance, the amplitude \( \tilde{c} \) in Eq. (16) depends on \( \omega \) and rapidly decreases when \( \omega \to \infty \). The second difference is that \( \cos \tilde{x} \) is substituted by \( \cos(2\tilde{x}) \). This means that the two-photon process is substituted by a four-photon process. In other words, the scattering occurs for the momentum

\[
p_l = \omega + \hbar l
\]

instead of Eq. (15) in the case of the primary nonlinear resonance. [Two diffraction orders with \( l = \pm 1 \) and \( l = \pm 2 \) are clearly seen in Fig. 2(b).] One further remark concerning Fig. 2(b) should be of interest. In our numerical experiment the scattering process is simulated for a coherent incident wave packet. The coherence of the wave packet does not play any role if one of the states involved in the Bragg scattering is initially empty [as it is in the case of Figs. 2(a) and 2(c)]. It influences the results, however, if both of the states \( p_l \) and \( p_{l'} \) are not empty [as it is in the case of Fig. 2(b)]. In the case of a completely incoherent incident beam, the modification of the initial distribution is less pronounced than in Fig. 2(b).

### C. Landau-Zener transitions

As shown in the preceding subsection, the scattering by a dynamical lattice (8) in the quasiregular case \( (K \ll 1) \) resembles very much the scattering by a rigid grating (3). There is also a difference in principle, however, due to a difference between the (quasi)energy spectrum of a quantum quasiregular system (its classical counterpart has only local integrals of the motion) and the spectrum of a regular system (the integrals are global). Namely, almost all crossings of the (quasi)energy levels are in fact avoided crossings with extremely small gaps (with the exception of crossings of the levels belonging to different symmetry classes). This feature of a quasiregular system leads to a specific quantum effect, which we discuss in this section in relation to Bragg scattering.

The basis for a formal analysis of the problem is the Schrödinger equation in the adiabatic representation [18]

\[
\dot{a}_m = \hbar \sum_{m \neq n} \frac{\phi_m}{E_m - E_n} \exp \left[ \frac{i}{\hbar} \int_{-\infty}^{t} (E_m - E_n) dt \right] a_n.
\]

Equation (18) is usually discussed for a system with a discrete energy spectrum. However, it can be also adopted for the present case of a system with a continuous spectrum (more precisely, with a band spectrum) because the laser field couples only the discrete set of states with the same Bloch vector. Then \( E_n = E_n(\epsilon) \) and \( \phi_m = \phi_m(\epsilon) \) denote the instantaneous Bloch specific quasienergies and eigenfunctions of the system (8), the overdot denotes the time derivative, and the \( a_n \) are the expansion coefficients of the wave function in the instantaneous basis: \( \psi = \sum_n a_n \phi_n \). The adiabatic regime corresponds to the solution \( a_n(t) = \text{const} \), which is realized if

\[
\frac{\hbar}{|E_m(\epsilon) - E_n(\epsilon)|} \ll 1.
\]

For sufficiently small \( \epsilon \sim \tau^{-1} \), the condition (19) can be met everywhere except at the locations of the avoided crossings \( E_n(\epsilon^*) = E_m(\epsilon^*) \), \( \epsilon^* = \epsilon(t^*) \). Therefore, one should analyze a single quasicrossing in more detail. In the vicinity of such a quasicrossing the level spacing is given by the eigenvalues of a \( 2 \times 2 \) matrix of the form

\[
A = \begin{pmatrix} \beta_1(t - t^*) & \Delta/2 \\ \Delta/2 & -\beta_2(t - t^*) \end{pmatrix},
\]

where \( \beta_1 \sim \beta_2 \sim \epsilon \) and \( \Delta \) is the width of the gap. In the same approximation the system wave function can be represented as \( \psi(t) = a_1(t) \phi_1(t) + a_2(t) \phi_2(t) \), where \( \phi_1(t) \) and \( \phi_2(t) \) are the eigenfunctions of the matrix (20). The initial condition is \( |a_1(t^* - \infty)| = 1, |a_2(t^* - \infty)| = 0 \). There are two limiting cases depending on the value of \( \epsilon \): for \( \Delta^2/\hbar \epsilon \gg 1 \) one has an adiabatic transition and

\[
|a_1(t^* + \infty)|^2 \approx 1, \quad |a_2(t^* + \infty)|^2 = \exp(-\pi \Delta^2/\hbar \epsilon \epsilon).
\]
for $\Delta^{2}/\hbar \dot{\epsilon} \ll 1$ one has a diabatic transition and
\[ |a_{1}(t^{*} + \infty)|^{2} \approx 2 \pi \Delta^{2}/\hbar \dot{\epsilon}, \quad |a_{2}(t^{*} + \infty)|^{2} \approx 1 \quad (22) \]
(for more details see the Appendix). In the intermediate case, the probabilities of level population are exchanged in a rather complicated way.

The key point of the scattering process discussed in Sec. III B is that the system follows the regime of diabatic transitions at the avoided crossings. This regime can be considered as an imitation of the adiabatic regime for a regular system. To validate it, the rate $\dot{\epsilon}$ should be small enough to meet the adiabatic condition for a regular system, but not too small in order to meet the condition for the diabatic transition at the avoided crossings. In the next paragraph we illustrate this statement by a more thorough numerical analysis of the scattering process.

To simplify the problem, we consider the diffraction of a plane wave with momentum $p_{in}$ satisfying the Bragg condition $p = \hbar l/2$ (we chose $l = 10$). Figures 3(a) and 3(b) compare the result of scattering in the case of a weak [$\omega = 2.9$, $K(\omega) \approx 1$] interaction of the resonances (quasiregular case, shown as a solid line) with that in the case of no interaction (regular case, shown as a dotted line). The latter case was simulated on the basis of the effective Hamiltonian $H_{eff} = p^{2}/2 - (\epsilon/2)\cos x$, which formally corresponds to the choice $\omega = \infty$ in Eq. (9). It is seen that in the case of the regular dynamics only states with $p = p_{f} = \pm 0.5$ are considerably populated. (The population of the neighboring states is a quantitative characteristic of the adiabatic approximation. It holds with an accuracy higher than $10^{-6}$ for $\tau = 64\pi$ and higher than $10^{-12}$ for $\tau = 128\pi$. In the quasiregular case ($\omega = 2.9$), a number of additional peaks appear, which correspond to the states populated by Landau-Zener transitions. We would like to stress that these transitions occur because of a violation of the diabatic condition (22), but not the adiabatic condition (21). In fact, when we increase $\tau$, the heights of the peaks increase, not decrease, as they would in the case of an almost adiabatic passage of the quasicrossings. We also note that, because the system has an infinite number of avoided crossings (with arbitrarily small gaps), one can never realize the pure adiabatic regime, whatever large values of $\tau$ are chosen. In other words, we can discuss only "an imitated adiabatic regime."

D. Chaotic scattering

In this section we briefly discuss case (ii), where the classical atom enters the chaotic regime for $\epsilon(t) > \epsilon_{cr}$. The key point of the problem is that neither adiabatic nor imitated adiabatic regimes can exist in this case in principle; there are no action integrals that could be conserved. Thus the final momentum of the classical atom does not correlate with its initial momentum, but takes a random value. The distribution of the random output depends on the overlap parameter (10) and the passage time $\tau$ in a rather complicated way. In the limit of large $\tau$, however [large in comparison to the characteristic period of the system dynamics given by the frequency (2)], the dependence on $\tau$ disappears and the distribution can be derived from the invariant distribution of the chaotic component for $K = K(\epsilon = 1)$. For example, in the case $\omega = 0.75$ [see Fig. 1(b)] the momentum of the outgoing atom takes random values in the interval
\[ |p_{f}| < I_{\max}/2, \quad (23) \]
where $I_{\max}$ is the action integral (5) for the invariant curve closest to the chaotic component.

In the quantum case, the situation resembles classical chaotic scattering. A plane-wave state $p_{in}$ is scattered into a number of states
\[ p_{f} = p_{in} + \hbar l, \quad (24) \]
with the only restriction imposed by Eq. (23) [see Fig. 3(c)]. Such a behavior of the quantum system can be explained by the mechanism of Landau-Zener transitions. As mentioned above, the main feature of a nonintegrable quantum system is the presence of many avoided crossings. In the case of quasiregular dynamics, the gaps of all avoided crossings are extremely small. This leads to the imitated adiabatic regime in the almost integrable systems [the condition for the diabatic transition (22) is satisfied]. When the classical counterpart of a nonintegrable quantum system is fully chaotic, however, the gaps are distributed according to the Wigner distribution [3,4], which suppresses small gaps and favors gaps of larger

FIG. 3. Probabilities of scattering of the plane wave $p_{in} = 0.5$ in the cases of regular (dotted lines), quasiregular [solid lines in (a) and (b)], and chaotic (c) dynamics. The parameters are (a) $\omega = 2.9$ and $\tau = 64\pi$, (b) $\omega = 2.9$ and $\tau = 128\pi$, and (c) $\omega = 0.75$ and $\tau = 128\pi$. 

\[ \end{eqnarray} \]
IV. CONCLUSION

We have considered Bragg scattering of an atomic beam by an amplitude-modulated standing laser wave, which acts as a dynamical grating. In the case of high modulation frequency ($K \ll 1$), the positions of the main diffraction peaks (we neglect here the states populated by the Landau-Zener transitions) are predicted by Eqs. (7), (15), and (17). An important feature of the scattering by a dynamical grating is that the positions of the peaks depend on the modulation frequency. This might find applications in the field of atomic optics since one can obtain a plane atomic wave (from a generally incoherent incoming atomic beam) with a tunable wave number.

In the case of low modulation frequency ($K \gg 1$), the system shows a chaotic regime of scattering. Namely, independently of the ratio of the atomic wave number to the field wave number (integer or not), a plane wave state is scattered into a large number of states [Eqs. (23) and (24)]. In analogy to other quantum chaotic systems, it is expected that the scattering probabilities are (quasi)random. The analysis of this regime, which is of primary interest for the field of quantum chaos, is planned to be the subject of a future work.

We also would like to note that, instead of the modulation of the laser intensity, modulation of the laser frequency can be used to observe the quasiregular and chaotic Bragg scattering. This case will involve the Hamiltonian of the form

$$H = p^2/2 - \varepsilon(t)[1 - \varepsilon_0 \cos(\omega t)] \cos x,$$

where $\varepsilon(t) = \exp[-(t/\tau)^2]$, \hspace{1cm}$\varepsilon(t)$ is proportional to the amplitude of the laser frequency modulation. For fixed $\varepsilon$ the classical and quantum dynamics of the system (25) were studied in Ref. [20]. A particular feature of this system is the superiority of the secondary resonances over the primary resonances.

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APPENDIX

To find the transition probability in the adiabatic case, we shall use the method of complex time [19]. According to Eq. (20), the quasienergies $E_{1,2}$ are determined by the equation

$$[\beta_1(t-t^*) - E_{1,2}([\beta_2(t-t^*) + E_{2,1} + \Delta^2/4 = 0].$$

Assuming for simplicity $\beta_1 = \beta_2 = \beta$ and $t^* = 0$ we have $E_{1,2} = \pm [(\beta t)^2 + (\Delta/2)^2]^{1/2}$. The initial condition corresponds to the asymptotic solution

$$a_1(t) = \exp[-i \int_0^t E_1(t') dt'], \hspace{1cm} t \to -\infty. \hspace{1cm} (A1)$$

We obtain the asymptotic value of $a_2(t)$ for $t \to +\infty$ by integrating Eq. (A1) in the complex plane, where the integration contour passes above the branching point $t_b = i\Delta/2\beta$. Deforming the contour in an appropriate way we find

$$a_2(+\infty) = \exp[-2i \int_0^t (\Delta^2/4 - \beta^2 t^2)^{1/2} dt]$$

$$= \exp[-\pi \Delta^2/8 \hbar \beta]. \hspace{1cm} (A2)$$

The probability of a diabatic transition can be calculated from perturbation theory, where the off-diagonal element of the matrix (20) is considered as a small parameter. The wave function

$$|\psi(t)\rangle = \left(\begin{array}{c} \exp[-i \int_0^t (\beta t') dt'] \\ 0 \end{array}\right) \hspace{1cm} (A3)$$

corresponds to zero transition probability ($\Delta = 0$). Substituting Eq. (A3) into the right-hand side of the Schrödinger equation with the Hamiltonian (20), we find

$$|a_2(+\infty)| \approx \frac{\Delta}{2\hbar} \int_{-\infty}^{\infty} \exp\left[-i \frac{2\beta t^2}{\hbar} \right] dt = \left(\frac{2\pi \Delta^2}{\hbar \beta}\right)^{1/2}. \hspace{1cm} (A4)$$

[11] Another experimental realization of the system of three interacting resonances was reported in the paper by J. C. Robinson, C. Bharucha, K. W. Madison, F. L. Moore, B. Sundaram, S. R.


[15] This condition is actually overestimated. The numerical simulation has shown that the integral (5) is fairly well conserved for $\tau \Omega_{\pi} = 4 \pi$.

[16] Criterion (10) is a slight modification of the Chirikov’s overlap parameter [7].

[17] We note that the width of the quantum nonlinear resonance is typically larger than the width of the classical resonance and they coincide only in the limit $\hbar \to 0$ [8].

